**S.No. 8048**

**24DPMA03**

(For the candidates admitted from 2024–25 onwards)

M.Sc. DEGREE EXAMINATION, AUGUST 2025

First Semester

Maths

ORDINARY DIFFERENTIAL EQUATIONS

Time : Three hours

Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Find all solutions of the equation $y'' + y' - 2y = 0$.

2. State the existence theorem for solutions of a second order initial value problem, with constant coefficients.

3. Define Wronskian of $n$ functions $\phi_1, \phi_2, \cdots \phi_n$.

4. Define linearly independent homogeneous equation of order $n$.

5. Write the Riccati equation.

6. Find the value of the Legendre polynomial $P_2(x)$.

7. Define regular singular point.

8. Give a note on indicial equation.

9. Find the solution of $y' = 3y^2/3$.

10. Define Lipsclitz constant.

PART B — ($3 \times 5 = 15$ marks)

Answer any THREE questions.

11. Let $\phi_1, \phi_2$ be two solutions of $L(y) = 0$ on an interval I, and let $x_0$ be any point in $I$. Then show that $\phi_1, \phi_2$ are linearly independent on I if and only if $W(\phi_1, \phi_2)$ $(x_0) \neq 0$.

12. Find all real-valued solutions of the equation $y^{(4)} + y = 0$.

13. Find the basis for the solution of the equation $y'' - \frac{2}{x^2} y = 0$ on $0 < x < \infty$.

14. Discuss the Bessel function of zero order of the second kind.

15. Prove the equation $y' = \frac{3x^2 - 2xy}{x^2 - 2y}$ is exact and solve.

PART C — $(5 \times 8 = 40$ marks)

Answer ALL the questions.

16. (a) Find all solution of the equation $y'' - y' - 2y = e^{-x}$.

Or

(b) If $\phi_1, \phi_2$ are two solutions of $L(y) = 0$ on an interval I containing a point $x_0$, then show that $W(\phi_1, \phi_2)(x) = e^{-a_1(x-x_0)} W(\phi_1, \phi_2)(x_0)$.

17. (a) Let $\phi_1, \cdots \phi_n$ be $n$ solutions of $L(y) = 0$ on an interval I containing a point $x_0$. Then prove that, $W(\phi_1, \cdots \phi_n)(x) = e^{-a_1(x-x_0)} W(\phi_1, \cdots \phi_n)(x_0)$.

Or

(b) Consider the equation with constant coefficients $L(y) = P(x)e^{ax}$, where $P$ is the polynomial given by $P(x) = b_0 x^m + b_1 x^{m-1} + \cdots b_m$, $(b_0 \neq 0)$. Suppose a is a root of the characteristic polynomial $p$ of $L$ of multiplicity $j$. Then prove that there is a unique solution $\Psi$ of $L(y) = P(x)e^{ax}$ of the form $\Psi(x) = x^j (c_0 x^m + c_i x^{m-1} + \cdots + c_m) e^{ax}$, where $c_0, c_1, \cdots c_m$ are constants determined by the annihilator method.

18. (a) If $\phi_1 \cdots \phi_n$ are $n$ solutions of $L(y) = 0$ on an interval $J$, prove that they are linearly independent there if, and only if, $W(\phi_1, \cdots \phi_n)(x) \neq 0$ for all $x$ in I.

Or

(b) State and prove the Existence theorem for analytic coefficients.

19. (a) Consider the second order Euler equation $x^2 y'' + axy' + by = 0$, $(a, b$ constants) and the polynomial $q$ given by $q(r) = r(r-1) + ar + b$. Prove that a basis for the solution of the Euler equation on any interval not containing $x = 0$ is given by $\phi_1(x) = |x|^{r1}, \phi_2(x) = |x|^{r2}$, in case $r_1, r_2$ are distinct roots of $q$, and by $\phi_1(x) = |x|^{r1}, \phi_2(x) = |x|^{r1} \log |x|$, if $r_1$ is a root of $q$ of multiplicity two.

Or

(b) Find the solution of the equation $L(y) = x^2 y'' + \frac{3}{2} xy' + xy = 0$, which has a regular singular point at the origin.

20. (a) Prove that the successive approximations $\phi_k$, defined by $\phi_0(x) = y_0$, exists as continuous function on $J : |x - x_0| \leq \alpha = \min imum \left\{ a, \frac{b}{M} \right\}$, and $(x, \phi_k(x))$ is in $R$ for $x$ in I. Indeed, the $\phi_k$ satisfy $|\phi_k(x) - y_0| \leq M |x - x_0|$ for all $x$ in I.

Or

(b) State and prove the existence theorem for convergence of the successive approximation.

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